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CHARACTERIZATION OF A CLASS OF MINIMAL RIGHT IDEALS OF LOOP-HALF-GROUPOID NEAR-RING OF TRANSFORMATIONS

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ABSTRACT

The study of near-ring of transformations was initiated by D.Ramakotaiah and G.KoteswaraRao [2]. In their paper they characterized a class of maximal and minimal right ideals. The study of loop-near rings was initiated by D.Ramakotaiah and Santakumari [4]. The study of loop-half-groupoid near rings was initiated by D.Ramakotaiah and PrabhakarRao [3]. In this paper we continue the study of loop-half-groupoid near-rings.

This paper is divided into three sections. In the first section, we present some basic definitions of loop-half-groupoid near-rings and some basic results without proofs. In second section we present some basic results without proofs which are necessary for our main work. In the third section we characterize a class of minimal right ideals of a loop-half-groupoid near-rings of transformations of a loop.

I. INTRODUCTION

For the definitions of half-groupoids, groupoids, loops, sub loops and normal sub loops see [5]. We begin this section with the following.

Definition 1.1

A system $N = (N, +, \cdot, o\ )$is called a loop-half-groupoid near-ring provided

(i)$N = (N, +, o\ )$is a loop.

(ii)$N = (N,\ \cdot\ )$is a half-groupoid.

(iii) $(a \cdot b) \cdot c = a \cdot (b \cdot c)$for all $a, b, c \in N$for which $a \cdot b,\ \ b \cdot c,\ \ a \cdot (b \cdot c), (a \cdot b) \cdot c$ are defined in N.

(iv) $a \cdot (b + c) = a \cdot b + a \cdot c$for all $a, b, c \in N$ for which $a \cdot (b + c),\ \ a \cdot b\ \ and\ a \cdot c$

are well defined in N

(v) $a \cdot o\ and\ o \cdot a \epsilon N$and $a \cdot o = o \cdot a = o$.

Remark1.2

For any 'a' belonging to an additive loop, we shall denote the unique left and right inverses of 'a' by $a_l\ and\ a_r$ respectively. It can be easily verified that $(a \cdot b)_r = a \cdot b_r$and $(a \cdot b)_l = a \cdot b_l$for all $a, b \in N$for which $a \cdot b, a \cdot b_l$ and $a \cdot b_r$are defined. We write

$a \cdot b$as .

Example 1.3

Every loop near-ring is a loop-half-groupoid near-ring..

Example 1.4

Let $(G, +, \bar{o}\ )$be an additive loop where $\bar{o}$ is the additive identity element of G. Let Δ be proper subset of Gcontaining $\bar{o}$. Define $a \cdot b = b$ for $\bar{o} \neq a \in \Delta\ and\ b \in G$. Define $\bar{o} \cdot b = \bar{o}$ and$a \cdot \bar{o} = \bar{o}$for all $a, b \in G$, then $(G, +, \cdot, \bar{o}\ )$is a loop-half-groupoid near-ring.

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Definition 1.5
Let $(N,+,\cdot,o)$be a loop-half-groupoid near-ring and let $(G,+,\bar{o})$be a loop, then G is called a N-loop provided there exists a mapping $(g,n) \to gn$ of $G \times N$ *into* G such that $g(n_1+n_2) = gn_1 + gn_2$ *and* $g(n_1 n_2) = (gn_1)n_2$for all$n_1, n_2 \in N$ *and* $g \in G$ for which $n_1 \cdot n_2$ is defined in N.

Definition 1.6
Let N be a loop-half-groupoid near-ring. Let G_1 *and* G_2 be N-loops. A homomorphism $f: G_1 \to G_2$is called a N-homomorphism provided $(g_n)f = (gf)n$ for all $g \in G$and $n \in N$. The kernel of f is called a N -kernel of G_1.

Definition 1.7
Let N be a loop-half-groupoid near-ring. An N-loop G is said to be an irreducible N-loop if it has no non-trivial N – kernels.

Lemma 1.8
If N is a loop-half-groupoid near-ring then a non-empty subset $M$of a N-loop is a N -kernel of G iff M is a normal subgroup of G.

Definition 1.9
A non-empty subset L of a loop-half-groupoid near-ring N is called a right ideal of Nprovided $(L,+,o)$ is a normal sub loop of N and $(l+n_1)n_2 + n_1 n_2 \in L$for all
$l \in L, n_1, n_2 \in N$ for which $(l+n_1)n_2$, $n_1 n_2$ are defined.

Definition 1.10
Let N be a loop-half-groupoid near-ring. Let G be an N-loop. An element $g \in G$ is called an N-generator of G or simply a generatorof G provided $g^N = G$.

Definition 1.11
If N is a loop-half-groupoid near-ring, then
(i) An irreducible N -loop with a generator is called an N –loop of type 0.
(ii) A N-loop of type 0 is called a N -loop of type 1 provided $g^N = G$ *or* $g^N = \{o\}$ for all $g \in G$.
(iii)A N-loop of type 1 is called a N -loop of type 2 if each non-zero element is a generator.

Definition 1.12
If N is a loop-half-groupoid near-ring, then any right ideal of N is said to be semi large if it has nonzero intersection with any one of the direct summand of N where N is written as a direct sum of right ideals.

II. PRELIMINARIES

In this section we present some basic definitions and basic results without proofs which are needed for our main work. All these definitions and results can be seen in [3].
We begin this section with the following:

Definition 2.1
Let $(G,+,\bar{o})$ be a loop and Δ be a subset of G. A set S of endomorphisms of G is called a Δ-centralizer of G provided:
(i) The zero endomorphism $\hat{o} \in S$.
ϕ(iii) $\Delta\emptyset \subseteq \Delta$ for all $\emptyset \epsilon S$.
(iv) For$\emptyset, \psi \epsilon S$ and $(\omega)\phi = (\omega)\psi$ for some $\bar{o} \neq \omega \epsilon \Delta \Rightarrow \Phi = \psi$.

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Definition 2.2
Let $(G,+,\bar{o}\ )$ be a loop and Δ be a subset of G and S be Δ-centralizer of G.
A mapping T of G into itself is called a Δ-centralizer of G over S provided $(\omega\phi)T=(\omega T)\phi$ for all $\omega\epsilon\Delta$ and $\phi\epsilon S$.

Remark 2.3
If $\bar{o}\epsilon\Delta$ and T is a Δ-transformation of Gover S, then T fixes $\bar{o}$. We shall denote the set of all Δ-transformations of G over S by $N(S,\Delta)$. It can be verified that for any endomorphism $\phi\ of\ G,(g\phi)_r=g_r\phi\ and\ (g\phi)_l=g_l\phi\ for\ all\ g\epsilon G.$

Lemma 2.4
Let $(G,+,\bar{o}\ )$ be a loop and Δ be a subset of G containing $\bar{o}$ and S be Δ-centralizer of G. Then $N(S,\Delta)$ is a loop-half-groupoid near-ring under the usual addition and iteration of mappings.

In general $N(S,\Delta)$ is not a loop-near-ring. We now state two sufficient conditions under which $N(S,\Delta)$is a loop-near-ring.

Lemma 2.5
$N(S,\Delta)$is a loop-near-ring under any one of the following conditions.
(i) for each $T\ in\ N(S,\Delta)$, $\Delta T\subseteq\Delta$.
(ii)for each $\omega\epsilon G,(\omega T)\phi=(\omega\phi)T\ for\ all\ \ T\ in\ N(S,\Delta)$ and $\phi\ \ in\ \ S$.

Throughout this remaining section we assume that $G$is a loop ,Δ a subset of G containing $\bar{o}$ properly and S be Δ-centralizer of G. $N(S,\Delta)$is the set of all Δ-transformations of G over S and $N(S,\Delta)$ is a loop-half-groupoid near-ring.

Lemma 2.6
Let Gbe loop and Δ a subset of G containing $\bar{o}$. Let S be Δ-centralizer of G then every non zero element of Δ is a $N(S,\Delta)$ generator of G .

Lemma 2.7
Let G be a loop and S be a set of endomorphisms of G containing $\bar{o}$such that S-$\hat{o}$ is a group of automorphismsof .
Then S is a centralizer of some subset $\bar{o}$ of G containing non zero element of G iff $\cup F(\emptyset)\neq G,\emptyset\in S$-$\hat{o}$, $\emptyset\neq I$, where I is the identity mapping of G and $F(\emptyset)=\{x\in G:x\emptyset=x\}$. If this is the case then G has a $N(S,\Delta)$ generator.

Definition 2.8
Let G be a loop, Δ a subset of Gcontaining $\bar{o}$ and S a Δ-centralizer of G.
Let $\bar{o}\neq\omega_1,\omega_2\epsilon\Delta$. Then $\omega_1\ and\ \omega_2$ are said to be S-equivalent if there exists $\emptyset\in S$-$\hat{o}$ such that $\omega_1\emptyset=\omega_2$.

Definition 2.9
The relation "S-equivaqlent" is an equivalence relation on Δ. If Γ is any subset of G, then we denote the set $\{n\epsilon N(s,\Delta):(\gamma)n=\ \bar{o}\ for\ all\ \gamma\epsilon\ \Gamma\}$ by $A(\Gamma)$. It can be seen that $A(\Gamma)$is a loop. If $N(S,\Delta)$is a loop-near-ring then $A(\Gamma)$ is a $N(S,\Delta)$ -loop.

Lemma2.10
containing ω. In particular if$N(S,\Delta)$ is a loop-near-ring then G is $N(S,\Delta)$ isomorphic to $A(G-\Gamma)$.

Theorem 2.11
If $N(S,\Delta)$ is a loop-near-ring, then G is a $N(S,\Delta)$ - loop of type $'o'$ if and only if for some S-equivaqlence classΓ, $A(G-\Gamma)$ does not contain a non zero nilpotent right ideal of nilpotency 2.

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Theorem 2.12
For each proper $N(S,\Delta)$-kernel G_1 of G and for each $\bar{o} \neq \omega\epsilon\Delta$, $\omega + G_1 \subseteq \Gamma$ where Γ is the S-equivaqlence containing ω.

Theorem 2.13
Let G be a loop. Let Δ be a subset of G containig $\bar{o}$and S be Δ-centralizer of G. If ω is a non zero element of Δ , then there exixts$T\epsilon\ N(S,\Delta)$ which maps every element of the
S-equivaqlence containing ω onto itself and maps every other element onto $\bar{o}$.

III. CHARACTERIZATION OF MINIMAL RIGHT IDEALS

In this section we characterize a class of minimal right ideals and a class of maximal right ideals of a loop-half-groupoid near-ring of Δ-transformations of a loop G over a set of endomorphisms of G

Throughout this section we assume that G is any loop and S is a set of endomorphisms of Gsuch that S-$\hat{o}$ is a group of automorphisms of G where $\hat{o}$ is the zero endomorphism of G. Also we assume that $\Delta$is a subset of G containing $\bar{o}$ and $N(S,\Delta)$ stands for the loop-half-groupoid near-ring of Δ-transformations of G over S which acts o-primitively on G as $N(S,\Delta)$-loopoid.

Lemma 3.1
If H is any subset of G, then the set $A(H) = \{T\epsilon\ N(S,\Delta): (h)T = \bar{o}$ for all hϵH $\}$ is a $N(S,\Delta)$ -loopoid.
Proof: Clearly $A(H)$ is a subloopof $(\ N(S,\Delta),+\ )$. Hence it is a loop.
Let $T'\epsilon N(S,\Delta)$and let $T\epsilon A(H)$ such that TT' is defined.
For any $h\epsilon H, (h)TT' = (hT)T' = (\bar{o})T' = \bar{o} \Rightarrow TT' \in A(H)$
Also for any $T \in A(H)$ and $T_1, T_2 \in N(S,T), T(T_1 + T_2) = TT_1 + TT_2$ and $T(T_1 T_2) = (TT_1)T_2$
Where $T(T_1 + T_2), TT_1, TT_2, T(T_1 T_2), (TT_1)T_2$are defined.
Therefore $A(H) = \{T\epsilon\ N(S,\Delta): (h)T = \bar{o}$ for all hϵH $\}$ is a $N(S,\Delta)$ -loopoid.

Lemma 3.2
If L is a minimal right ideal of $N(S,\Delta)$ such that L is not contained in $A(\Delta)\ then\ L\ isN(S,\Delta)$ – loopoidhomomorphicto G.
Proof: Since $L \not\subset A(\Delta)$,there exists an element $\bar{o} \neq \omega\epsilon\Delta$ such that $\omega L \neq \{\bar{o}\}$. Since ω is a $N(S,\Delta)$ generator of G, we have $G = \omega N(S,\Delta)$. Clearly ωL is a subloop of G.Since L is a normal subgroup of G, we have that ωL is also a normal subgroup of G.

Let $\omega T_1 \in \omega L$ and $g \in G = \omega N(S,\Delta) \Longrightarrow g = \omega T'$ for some $T' \in N(S,\Delta)$. Let $T \in\ N(S,\Delta)$
Such that $(T_1 + T')T$ and $T'T_r$ are defined.

Now $(\omega T_1 + \omega T')T + \omega T'T_r = \omega[(T_1 + T')T + T'T_r\ ]\epsilon\omega L$. Therefore ωL is a $N(S,\Delta)$ –loopoid kernel of G.Since G is irreducible and $L \neq \{\bar{o}\}$, we have $\omega L = G$. Now define a mapping $\emptyset: L \to G\ by\ \emptyset(l) = \omega l\ for\ all\ l \in L$. Clearly $\emptyset$ is $N(s,\Delta)$-loopoid epimorphism of L onto G. Also clearly ker$\emptyset$ is a right ideal of $N(s,\Delta)$ which is properly contained in L. Since L is a minimal right ideal, we have ker$\emptyset$=$\{\hat{o}\}$ and hence $\emptyset$ is one-one. Hence $\emptyset$ is an $N(s,\Delta)$-loopoid isomorphism of $L\ onto\ G$.

Theorem 3.3
Let G be any loop and $\{\bar{o}\} \neq \Delta \subseteq$ G. Let $S\ and\ S'$ be two Δ-centralizers of G such that $S\ \subseteq S'$. Then $N(S,\Delta) = N(S',\Delta)$ if and only if $= S'$.
Proof: If $S = S'$ then there is nothing to prove.
Conversly suppose that $N(S,\Delta) = N(S',\Delta)$, suppose if possible $S \neq S'$.
Since $S\ \subseteq S'$, there exists $\emptyset' \in S'$ such that $\emptyset' \in S$, clearly $\emptyset' \neq \hat{o}$.
Let ω be any non zero element of Δ. Let $C\ and\ C'$ be respectively $S\ and\ S'$ equivalence classes containing ω.

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Now $C = \{\omega\emptyset : \emptyset \in S - \hat{o}\}$ *and* $C' = \{\omega\emptyset : \emptyset \in S' - \hat{o}\}$.
We have $\omega\emptyset' \in C'$. Suppose if possible $\omega\emptyset' \in C$. Then there exists $\emptyset \in S - \hat{o}$ such that $\omega\emptyset' = \omega\emptyset$.
Since $S \subseteq S'$, we have $\emptyset \in S'$. Now $\emptyset$ *and* $\emptyset'$ are elements of $S' - \hat{o}$ such that $\omega\emptyset' = \omega\emptyset$ where $\bar{o} \neq \omega\epsilon\Delta$. By the definition of Δ-centralizer, $\emptyset = \emptyset'$ which is a contradiction.
Therefore $\omega\emptyset' \notin C$. Write $\omega\emptyset' = \omega_1$.
By lemma 2.6 there exists a $T \in N(S, \Delta)$ such that $\omega T = \omega_1$ and T maps every element of G which does not belong to the $S-$ equivalence class C onto $\bar{o}$. Since $N(S, \Delta) = N(S', \Delta)$, we have $T \in N(S', \Delta)$ and hence $\bar{o} = \omega_1 T = (\omega\emptyset')T = (\omega T)\emptyset'$.
Since $\emptyset$ is an automorphism of G, it follows that $\omega T = \bar{o}$. Therefore $\bar{o} = \omega T = \omega_1 = \omega\emptyset'$.
Again since $\emptyset'$ is an automorphism. $\emptyset' = \bar{o}$, which is a contradiction. Therefore $S = S'$.

Corrolary 3.4
The set of all loop endomorphisms $\emptyset$ of loop G such that $(\omega\emptyset)T = (\omega T)\emptyset$ for all $\omega\epsilon\Delta, T \in N(S, \Delta)$ and $\Delta\emptyset \subseteq \Delta$ is S itself.
Proof:

Let $S' = \{\emptyset : \emptyset \textit{ is a loop endomorphism of } G \textit{ such that } \Delta\emptyset \subseteq \Delta \textit{ and } (\omega\emptyset)T = (\omega T)\emptyset$ for all $\omega\epsilon\Delta, T \in N(S, \Delta)\}$.
Now we shall prove that S' is a Δ-centralizer of G.
Clearly $\hat{o} \in S'$ and $\Delta\emptyset \subseteq \Delta$ for all $\emptyset \in S - \hat{o}$.
Let $\emptyset$ be a non zero element of S'. Since G is irreducible , the kernel of $\emptyset$ must be either G or $\{\ \bar{o}\ \}$. Since $\emptyset \neq \hat{o}$ it follows that ker$\emptyset = \{\ \bar{o}\ \}$ and hence $\emptyset$ is one-one.
Let $g \in G$ and $\omega\emptyset \neq \omega\epsilon\Delta$. Now $\omega\emptyset\epsilon\Delta$ and $\emptyset \neq \bar{o}$.
Hence by lemma 2.6, $\omega\emptyset$ is a $N(S, \Delta)$ -generator of G. Therefore, there exists a $T \in N(S, \Delta)$ such that $(\omega\emptyset)T = g$. Put $g_1 = \omega T$. Now $g_1 \in G$ *and* $g_1\emptyset = (\omega T)\emptyset = (\omega\emptyset)T = g$.
Hence $\emptyset$ is onto . Therefore $\emptyset$ is an automorphism of G.
Finally suppose that $\omega\emptyset = \omega\Psi$, where $\emptyset, \Psi \in S - \hat{o}$ and $\bar{o} \neq \omega\epsilon\Delta$.
Let $g \in G$. Then there exists a $T \in N(S, \Delta)$ such that $\omega T = g$.
Now $g\emptyset = (\omega T)\emptyset = (\omega\emptyset)T = (\omega\Psi)T = (\omega T)\Psi = g\Psi$. This is true for all $g \in G$.
Hence $\emptyset = \Psi$.
Therefore S' is a Δ-centralizer of G.
By the definition of S', $S \subseteq S'$. It can be easily verified that $N(S, \Delta) = N(S', \Delta)$.
Therefore by the above theorem 3.3 we have $S = S'$.

Lemma 3.5
Let C be an S-equivalence class on Δ. Then $A(G - C)$ is a $N(S, \Delta)$-loopoid of type 0 and hence it is a minimal right ideal of $N(S, \Delta)$.
Proof:
Clearly by lemma 3.1, $A(G - C)$ is a $N(S, \Delta)$-loopoid. Let $g \in G$.
By theorem 2.13 there exists a $T \in N(S, \Delta)$ such that $gT = g$ and $g'T = \bar{o}$ for all $g' \in G - C \Rightarrow T \in A(G - C)$.
Now let $g_1 \in C$.
Then $g_1 = g\emptyset$ for some $\emptyset \in S - \hat{o} \Rightarrow g_1 T = (g\emptyset)T = (gT)\emptyset = g\emptyset = g_1$.
Hence $g_1 T = g_1$ for some $g_1 \in C$. Now we shall show that $TN(S, \Delta) = A(G - C)$ where $TN(S, \Delta) = \{TT_1 = T_1 \in N(S, \Delta)$ *and* TT_1 is defiHence ned$\}$.
Let $TT_1 \in TN(S, \Delta)$.
For any $g \in G - C, (g)TT_1 = (gT)T_1 = (\bar{o})T_1 = \bar{o}$.
Hence $TT_1 \in A(G - C)$.
Conversly suppose that $T_1 \in A(G - C)$.
Define $T_2 : G \rightarrow G$ by $(g)T_2 = (g)T_1$ if $g \in C$ *and* $\bar{o}$ if $g \in G - C$.
Now it can be easily verified that $T_2 \in N(S, \Delta)$ and $T_1 = TT_2 \Rightarrow T_1 \in TN(S, \Delta)$.
Therefore $TN(S, \Delta) = A(G - C)$.

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Put $K = A(G - C)$. Now K is a right ideal of $N(S, \Delta)$.

Further for any $g \in C, gK = gA(G - C) = g\,TN(S, \Delta) = gN(S, \Delta) = G$.
For some $g \in C$. Define $\emptyset g: K \to gK$ by $(k)\emptyset g = gk$ $for\ any$ $k \in K$.
Clearly $\emptyset g$ is a $N(S, \Delta)$-loopoidepimorphism of Konto G. Since S-equivalent elements have equal annihilators, we have $A(G) = A(C)$.
Therefore ker$\emptyset g = K \cap A(g) = A(G - C) \cap A(C) = A(G) = \{\hat{o}\}$
Therefore $\emptyset g$ is an $N(S, \Delta)$-loopoid isomorphism of K $onto$ G. Since G is a $N(S, \Delta)$-loopoid of type 0, $K = A(G - C)$ is also a $N(S, \Delta)$-loopoid of type 0 and hence $A(G - C)$is a minimal right ideal of $N(S, \Delta)$.

Theorem 3.6
Let G be a N-loopoid of type 0. If g is a N -generator of G, then $A(g)$ is a maximal right ideal of N.
Proof:
Since g is a N -generator of G, we have $gN = G$.
Define a mapping $\emptyset: N^+ \to G$ by $\emptyset(x) = gx$ $for\ all$ $x \in N^+$.
For any $x_1, x_2 \in N^+, \emptyset(x_1 + x_2) = g(x_1 + x_2) = gx_1 + gx_2 = \emptyset(x_1) + \emptyset(x_2)$.
For any $x \in N^+, n \in N, \emptyset(xn) = g(xn) = (gx)n = \emptyset(x)n$.
Let $g_1 \in G \to g_1 = gx$ for some $x \in N$. Now $x \in N$ and $\emptyset(x) = gx$. Hence $x \in$ ker $\emptyset$ iff $\emptyset(x) = \bar{o}$ iff $gx = \bar{o}$ iff $x \in A(g)$.

Therefore, $\emptyset$ is a N-loopoid homomorphism of N^+ onto G with Kernel $A(g)$. Hence $N^+/A(G)$ is a N-loopoid isomorphic to G. Since $A(g)$ is the kernel of N-loopoid homomorphism, it is a right ideal of N. Since G is irreducible, we have $N^+/A(G)$ is also irreducible and hence $A(g)$ is a maximal right ideal of N.

Lemma 3.7
Let C be an S-equivalence class on Δ. Then $A(C)$ is a maximal right ideal of $N(S, \Delta)$.
Proof:
By the above theorem 3.6 ,$A(G)$ is a maximal right ideal of $N(S, \Delta)$ for any $g \in \Delta$. Since all the elements of an S-equivalence class have the same annihilators, we have $A(C) = A(g)$ for some $g \in C$. Hence $A(C)$ is a maximal right ideal of $N(S, \Delta)$. Hence the result.

Lemma 3.8
Let C be an S-equivalence class. Then $N(S, \Delta)$ is a direct sum of $A(C)$ and $A(G - C)$.
Proof:
WE have $A(G - C) \cap A(C) = A(G) = \{\hat{o}\}$.Since$A(G - C)$ is a minimal right ideal, it is a non zero right ideal of $N(S, \Delta)$ and hence $A(G - C) \nsubseteq A(C)$. Since $A(C)$ is a maximal right ideal, we have $A(C)$+ $A(G - C) = N(S, \Delta)$. Hence $N(S, \Delta)$ is a direct sum of $A(C)$ and $A(G - C)$.

Lemma 3.9
If L is a minimal right ideal of $N(S, \Delta)$such that L is not contained in $A(\Delta)$ and L is a semilarge , then $L = A(G - C)$ where C is an S-equivalence class of Δ.
Proof:

Suppose L is a minimal right ideal of $N(S, \Delta)$ such that L is not contained in $A(\Delta)$ and L is a semilarge .
Write $G_1 = \{g \in \Delta: gL \neq \{\bar{o}\}\}$. Since L is not contained in $A(\Delta)$we have at least one $g \in \Delta$such that $gL \neq \{\bar{o}\}$, therefore $G_1 \neq \emptyset$.
Let $g \in G_1 \Rightarrow gL \neq \{\bar{o}\}$.
Now for all $\emptyset \in S - \hat{o}$, $(g\emptyset)L = (gL)\emptyset \neq \hat{o}$, hence $(g)\ \emptyset \in G_1$.
Therefore the S-equivalence class Ccontaining g is contained in G_1. Thus G_1 contains an S-equivalence class Con Δ.
Assume that $\neq A(G - C)$. Since L and $A(G - C)$ are minimal right ideals , we have $L \cap A(G - C) = \{\hat{o}\}$. Since

$CL \neq \{\bar{o}\}$, we have $L \nsubseteq A(C)$. Since L is a minimal right ideal, it follows that $L \cap A(C) = \{\hat{o}\}$. By Lemma 3.7 we have $A(C)$ is a maximal right ideal $\Rightarrow L + A(C) = N(S, \Delta)$ where the sum is direct.
By Lemma 3.8 we have $A(C)$+ $A(G - C) = N(S, \Delta)$where the sum is direct.
Since L is semi large either $L \cap A(G - C) \neq \{\hat{o}\}$or $L \cap A(C) \neq \{\hat{o}\}$
But we have $L \cap A(G - C) = \{\hat{o}\}$ and $L \cap A(C) = \{\hat{o}\}$ which is a contradiction.
Therefore $L = A(G - C)$.

Theorem 3.10
Let L be a right ideal of $N(S, \Delta)$ such that $L \nsubseteq A(\Delta)$ and L be semi large. Then L is a minimal right ideal of $N(S, \Delta)$iff$L = A(G - C)$for some S-equivalence class C on Δ
Proof:
The proof follows from the lemmas 3.5 and 3.9.

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